Mathematics for Electrical Engineering Homework # 1

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Problem 1.1 Calculate the following $y(t)$, and describe them as simply as possible.

$$\begin{array}{ll}
\text{(i)} & y(t) = \int_{-\infty}^{\infty} \sigma e^{3\sigma} u(2t-\sigma) d\sigma \\
\text{(ii)} & y(t) = \int_{-\infty}^{\infty} u(3t-\sigma) u(\sigma - 4) d\sigma \\
\text{(iii)} & y(t) = \int_{-\infty}^{\infty} t^2 \delta(\frac{\sigma}{2}) e^{-\sigma} u(\sigma) d\sigma \\
\text{(iv)} & y(t) = \int_{-\infty}^{\infty} (t-\sigma) e^{-(t-\sigma)} u(t-\sigma) u(2\sigma) d\sigma \\
\text{(v)} & y(t) = \int_{-\infty}^{\infty} [\delta(2t-\sigma) - e^{-3(t-\sigma)} u(t-\sigma)] e^{-2\sigma} t^3 u(\sigma) d\sigma
\end{array}$$

SOLUTIONS

Problem 1.1 (i)

$$\begin{aligned}
y(t) &= \int_{-\infty}^{\infty} \sigma e^{3\sigma} u(2t-\sigma) d\sigma \\
&= \int_{-\infty}^{2t} \sigma e^{3\sigma} d\sigma \quad (\text{since } u(2t-\sigma) = 0,\ \sigma > 2t.) \\
&= \left[\frac{\sigma}{3} e^{3\sigma} - \frac{1}{9} e^{3\sigma} \right]_{-\infty}^{2t} \\
&= \frac{2t}{3} e^{6t} - \frac{1}{9} e^{6t}
\end{aligned}$$

(ii)

$$\begin{aligned}
y(t) &= \int_{-\infty}^{\infty} u(3t-\sigma) u(\sigma - 4) d\sigma \\
&= \begin{cases} \int_{4}^{3t} 1 d\sigma, & 3t \geq 4 \\ 0, & 3t < 4 \end{cases} \\
&= \left(\int_{4}^{3t} 1 d\sigma \right) u(3t-4) \\
&= [\,\sigma\,]_{4}^{3t}\, u(3t-4) \\
&= (3t-4) u(3t-4)
\end{aligned}$$

(iii)

$$\begin{aligned}
y(t) &= \int_{-\infty}^{\infty} t^2 \delta(\frac{\sigma}{2}) e^{-\sigma} u(\sigma) d\sigma \\
&= t^2 \int_{-\infty}^{\infty} \delta(\tau) e^{-2\tau} u(2\tau) 2 d\tau \quad (\text{Let } \tau = \frac{\sigma}{2} \text{ and integrate with respect to } \tau.) \\
&= 2t^2 \int_{-\infty}^{\infty} \delta(\tau) e^{-2\cdot 0} u(2\cdot 0) d\tau \\
&= 2t^2 \int_{-\infty}^{\infty} \delta(\tau) d\tau \\
&= 2t^2
\end{aligned}$$

(iv)

$$
\begin{aligned}
y(t) &= \int_{-\infty}^{\infty} (t-\sigma)e^{-(t-\sigma)}u(t-\sigma)u(2\sigma)d\sigma \\
&= \begin{cases} \int_0^t (t-\sigma)e^{-(t-\sigma)}d\sigma, & t \geq 0 \\ 0, & t < 0 \end{cases} \\
&= \left(\int_0^t (t-\sigma)e^{-(t-\sigma)}d\sigma\right) u(t) \\
&= \left(\int_0^t \tau e^{-\tau}d\tau\right) u(t) \quad \text{(Let } \tau = t-\sigma \text{ and integrate with respect to } \tau.) \\
&= [-\tau e^{-\tau} - e^{-\tau}]_0^t u(t) \\
&= (1 - te^{-t} - e^{-t})u(t)
\end{aligned}
$$

(v)

$$
\begin{aligned}
y(t) &= \int_{-\infty}^{\infty} [\delta(2t-\sigma) - e^{-3(t-\sigma)}u(t-\sigma)]e^{-2\sigma}t^3u(\sigma)d\sigma \\
&= t^3\int_{-\infty}^{\infty} [\delta(2t-\sigma)e^{-2\sigma}u(\sigma) - e^{-3(t-\sigma)}u(t-\sigma)e^{-2\sigma}u(\sigma)]d\sigma \\
&= t^3\int_{-\infty}^{\infty} \delta(2t-\sigma)e^{-4t}u(2t)d\sigma - \begin{cases} t^3\int_0^t e^{-3(t-\sigma)}e^{-2\sigma}d\sigma, & t \geq 0 \\ 0, & t < 0 \end{cases} \\
&= t^3\left(e^{-4t}\int_{-\infty}^{\infty} \delta(2t-\sigma)d\sigma - \int_0^t e^{-3t+\sigma}d\sigma\right) u(t) \\
&= t^3\left(e^{-4t} - e^{-3t}[e^{\sigma}]_0^t\right) u(t) \\
&= t^3(e^{-4t} - e^{-3t}(e^t - 1))u(t) \\
&= t^3(e^{-4t} - e^{-2t} + e^{-3t})u(t)
\end{aligned}
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