Mathematics for Electrical Engineering Homework # 2

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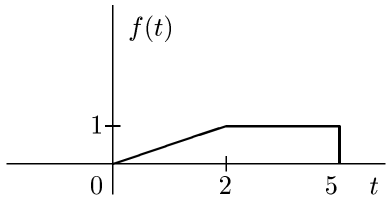
Problem 2.1 Calculate Laplace transforms $F(s)$ of following functions $f(t)$;
(i) $f(t) = t^2 e^{-3t}$
(ii) $f(t) = e^{-5t}\cos 2t$
(iii) $f(t) = (e^{-5t}\cos 2t)u(t-3)$
(iv) $f(t) = \int_0^t \tau \cos 5\tau d\tau$
(v)



SOLUTIONS
Problem 2.1 (i) Let $f_1(t) = e^{-3t}$. Then we have $F_1(s) = \dfrac{1}{s+3}$.

$$\begin{aligned} F(s) &= (-1)^2 \frac{d^2}{ds^2} F_1(s) \\ &= \frac{d^2}{ds^2}\frac{1}{s+3} \\ &= -\frac{d}{ds}\frac{1}{(s+3)^2} \\ &= \frac{2}{(s+3)^3} \end{aligned}$$

(ii) Let $f_1(t) = \cos 2t$. Then we have $F_1(s) = \dfrac{s}{s^2+2^2}$. Hence, we obtain

$$F(s) = F_1(s+5) = \frac{s+5}{(s+5)^2+4} = \frac{s+5}{s^2+10s+29}.$$

(iii) Since

$$\begin{aligned} f(t) &= (e^{-5t}\cos 2t)u(t-3) \\ &= e^{-15}e^{-5(t-3)}\cos[2(t-3)+6]u(t-3) \\ &= e^{-15}e^{-5(t-3)}[\cos 2(t-3)\cos 6 - \sin 2(t-3)\sin 6]u(t-3), \end{aligned}$$

we have

$$F(s) = \mathcal{L}[f(t)] = e^{-15}\cos 6\mathcal{L}[e^{-5(t-3)}\cos 2(t-3)u(t-3)] - e^{-15}\sin 6\mathcal{L}[\sin 2(t-3)u(t-3)] \qquad (1)$$

where

$$F_1(s) = \mathcal{L}[e^{-5t}\cos 2t] = \frac{s+5}{(s+5)^2+2^2},\ F_2(s) = \mathcal{L}[e^{-5t}\sin 2t] = \frac{2}{(s+5)^2+2^2}.$$

Therefore, we obtain from (1)

$$F(s) = \frac{e^{-3s-15}}{s^2+10s+29}\left((s+5)\cos 6 - 2\sin 6\right).$$

(iv) Let $f_1(t) = t\cos 5t$. Then we have

$$\begin{aligned} F_1(s) &= (-1)\frac{d}{ds}\frac{s}{s^2+5^2} \\ &= (-1)\frac{(s^2+25)-2s^2}{(s^2+25)^2} \\ &= \frac{s^2-25}{(s^2+25)^2}. \end{aligned}$$

By the rule of the Laplace transform for the integration, we obtain

$$F(s) = \frac{F_1(s)}{s} = \frac{s^2-25}{s(s^2+25)^2}.$$

(v) Since

$$\begin{aligned} f(t) &= \frac{1}{2}t[u(t)-u(t-2)] + [u(t-2)-u(t-5)] \\ &= \frac{1}{2}tu(t) - \frac{1}{2}(t-2)u(t-2) - u(t-5) \end{aligned}$$

we get

$$\begin{aligned} F(s) &= \mathcal{L}[\frac{1}{2}tu(t)] - \mathcal{L}[\frac{1}{2}(t-2)u(t-2)] - \mathcal{L}[u(t-5)] \\ &= \frac{1}{2s^2} - \frac{e^{-2s}}{2s^2} - \frac{e^{-5s}}{s} \\ &= \frac{1}{2s^2}(1-e^{-2s}-2se^{-5s}). \end{aligned}$$

For this problem, we may use the definition of the Laplace transform:

$$\begin{aligned} F(s) &= \int_0^2 \frac{1}{2}te^{-st}dt + \int_2^5 1\cdot e^{-st}dt \\ &= \frac{1}{2}\left[-\frac{1}{s}te^{-st} - \frac{1}{s^2}e^{-st}\right]_0^2 + \left[-\frac{1}{s}e^{-st}\right]_2^5 \\ &= \frac{1}{2}\left[-\frac{2}{s}e^{-2s} - \frac{1}{s^2}e^{-2s} + \frac{1}{s^2}\right] + \left[-\frac{1}{s}e^{-5s} + \frac{1}{s}e^{-2s}\right] \\ &= \frac{1}{2}\left(\frac{1}{s^2} - \frac{1}{s^2}e^{-2s} - \frac{2}{s}e^{-5s}\right) \end{aligned}$$