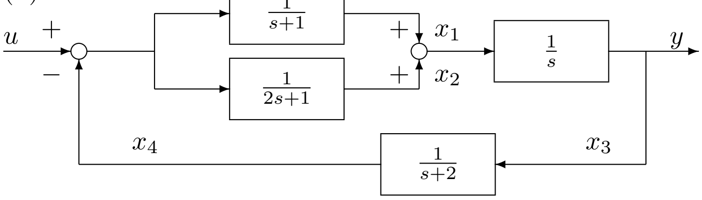
Control Problems

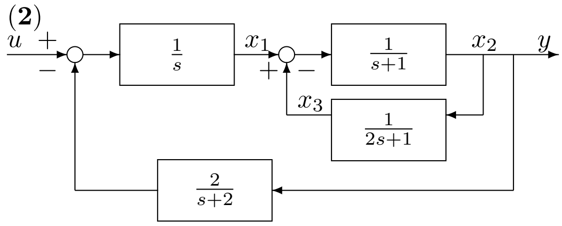
Jun Yoneyama

1.1 Find the transfer functions from u to y of the following systems depicted in figures below. Write down the state and output equations.

(1)



(2)



2.1 Find the eigenvalues and associated eigenvectors of the following matrices. Then diagonize them.

$$\textbf{(1)}\ A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & -3 \end{bmatrix} \textbf{(2)}\ A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \textbf{(3)}\ A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

2.2 Calculate e^{At}.

$$\textbf{(1)}\ A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -3 & 2 \\ 0 & -1 & 0 \end{bmatrix} \textbf{(2)}\ A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -2 & -1 \\ 0 & 0 & 2 \end{bmatrix}$$

2.3 Given the state and output equations

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u, \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix} x \end{aligned}$$

with the initial state and the input

$$x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} x,\ u(t) = 2$$

(1) Calculate $y(t)$.

(2) Let $y(\infty)$ be the steady state of $y(t)$. Calculate

$$J = \int_0^\infty (y(t) - y(\infty))^2 dt$$

2.4 Given the system

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x,\ x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

then calculate

$$J = \int_0^\infty x^T Q x dt$$

where

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

3.1 Check whether the following systems with (A, B, C) are controllable and/or observable.

$$\textbf{(1)}\ A = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{bmatrix},\ B = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix},\ C = \begin{bmatrix} 0 & 0 - 1 \end{bmatrix}$$

$$\textbf{(2)}\ A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 1 & 0 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},\ C = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

$$\textbf{(3)}\ A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix},\ B = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix},\ C = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

3.2 Given the system with the following matrices

$$A = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -2 & 0 \\ 0 & 0 & -1 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},\ C = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix},$$

(1) Check the controllability and observability.

(2) Consider the system

$$\begin{aligned} \dot{x} &= Ax \\ y &= Cx \end{aligned}$$

where the matrices A and C are given above. Then is it possible to choose the initial state $x(0)$ such that the output $y(t) = te^{t}$. If so, then find such an $x(0)$.

4.1 Transform the following systems into controllable and observable canonical forms, respectively.

$$\textbf{(1)}\ A = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{bmatrix},\ B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},\ C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

$$\textbf{(2)}\ A = \begin{bmatrix} -4 & 2 & 0 \\ 1 & -3 & 1 \\ 0 & 1 & -2 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

5.1 Check whether the following matrices are positive definite or negative definite.

$$\textbf{(1)}\ A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}\ \textbf{(2)}\ A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 1 \\ 1 & 1 & 3 \end{bmatrix}\ \textbf{(3)}\ A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 1 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

6.1 Find the state feedback low such that the poles of the resulting closed-loop system are $-4, -5$.

$$\textbf{(1)}\ \dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u,\ \textbf{(2)}\ \dot{x} = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

6.2 Find the state feedback low such that the poles of the resulting closed-loop system are $-2, -3 \pm j3$.

$$\textbf{(1)}\ \dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & -1 & -5 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u,\ \textbf{(2)}\ \dot{x} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} u$$

6.3 Design an observer such that the poles of the error systems are $-5, -6$.

$$\begin{aligned} \textbf{(1)}\ \dot{x} &= \begin{bmatrix} -1 & 0 \\ 1 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u, \\ y &= \begin{bmatrix} 0 & 1 \end{bmatrix} x \\ \textbf{(2)}\ \dot{x} &= \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u, \\ y &= \begin{bmatrix} 1 & 1 \end{bmatrix} x \end{aligned}$$

7.1 Given the system

$$\dot{x} = \begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u,$$

and the performance index

$$J = \int_0^\infty (x^T Q x + r u^2) dt.$$

Design the optimal regulators with the weighting matrices:

$$(\mathbf{1})Q = \begin{bmatrix} 1 & 0 \\ 0 & 8 \end{bmatrix}, \ r = 1, \ (\mathbf{2})Q = \begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix}, \ r = 1.$$