Spin–orbit interactions and suppression of dephasing in a lightly hydrogenated diffusive graphene

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The light hydrogenation of graphene introduces a spin–orbit interaction (SOI) caused by out-of-plane (OP) symmetry-breaking via the formation of an sp² distortion (the so-called Rashba-type SOI). It is known that the effective magnetic field \( B_{SOI} \) due to SOIs conventionally causes spin-flipping, resulting in dephasing of the electron waves in thin metals, semiconductor two-dimensional electron gases, and carbon nanotubes. Herein, we report that \( B_{SOI} \) of a Rashba-type SOI, introduced by precisely controlling the hydrogenation of graphene, does not cause spin-flipping and rather suppresses dephasing in phase-interference phenomena like weak localization (WL). The OP-symmetric \( B_{SOI} \), unique to graphene plane, suppresses dephasing for changes on temperature and the magnetic field in samples within the crossover hydrogenation regime \((0.03\% < N_H < 0.1\%)\), where \( N_H \) is the hydrogenation volume) between ballistic and diffusive charge transport.

SOIs in graphene are highly important with respect to two-dimensional (2D) topological insulating (TI) states, consisting of only small-mass atoms\(^{15}\). The possibility of correlating SOIs with other quantum phenomena in graphene has also attracted considerable attention\(^{6,8}\) (see SI.1). Phase interference between electron spin waves in 2D materials yields various attractive quantum phenomena within a diffusive charge-transport regime, in which electron waves are scattered but the phase memory is preserved [e.g., weak localization (WL), anti-WL, the Aharonov–Bohm and Altshuler–Aronov–Spivak effects, and the universal conductance fluctuation (see SI.2)\(^{9-12}\)]. The correlation of these phenomena with SOI is worth investigating. Phenomena arising from phase interference have been experimentally reported only in SOI-absent graphene with a large number of adatoms and ripples in the substrate\(^{13,14}\). An enhanced spin relaxation time, due to magnetic defects, and large-magnitude WL (i.e., exhibiting a resistance maximum due to constructive phase interference between two partial electron waves with the same phase when both waves encircle a 2D plane in opposite directions along time-reversal symmetry paths; SL.2) have been reported\(^{13}\).

On the other hand, a few recent theories have predicted that SOIs correlate with phase interference in graphene. They examined out-of-plane (OP) mirror-symmetric SOI systems of graphene\(^8\) with the Berry phase\(^7\). They showed that the SOIs suppress dephasing in WL correction. Because WL originates from a constructive phase interference between two electron spin waves with exactly the same phase and is highly sensitive to dephasing by its own effective magnetic field \( B_{SOI} \), one can examine presence of SOI through it. In conventional 2D materials, those with broken-OP-symmetric SOI systems, \( B_{SOI} \) causes spin-flipping and transforms WL to anti-localization (AL; showing a resistance minimum due to destructive phase interference of two electron waves with opposite phases) through a phase shift of \( \pi \) (SI.2). In peculiar OP-symmetric SOI systems, like graphene, \( B_{SOI} \) does not cause spin-flipping and suppresses dephasing\(^6\). Moreover, a Berry phase factor \((\beta \pi) < 0.6\) in the presence of SOIs yields suppression of WL because \( B_{SOI} \) (i.e., a random Rashba field) acts like a magnetic scattering center\(^7\). Correlations with electron–phonon interactions also considerably affect the physical properties related to SOIs\(^{16-18}\).

The coexistence of SOIs with phase interference in graphene is, however, difficult to realize experimentally. A heavy volume of adatoms (e.g., \( N_H \gg 0.1\% \)) forms a diffusive charge-transport regime, inducing phase interference while destroying SOIs. In contrast, a small \( N_H \) (e.g., \(< 0.05\%\)) results in a ballistic charge-transport regime, introducing SOIs while decreasing phase interference. Precise control of \( N_H \) is, thus, crucial to producing the crossover region between these two regimes. Only one previous publication has reported the plasma hydrogenation of graphene; a significant increase in the spin relaxation length (up to 7 \( \mu m \)) was observed due to Rashba-type SOIs for a sample with \( N_H \approx 0.02\%\).

In the present study, a hydrogen silsesquioxane (HSQ) resist \([\text{HSI}0_{32}]_n\) has been applied to mechanically exfoliated pristine graphene. When the HSQ resist is irradiated with an electron beam (EB), a H atom detaches from the resist and forms a C–H bond on the graphene surface. One can achieve an accurately low-level hydrogenation (\( N_H < 0.1\% \) within 0.01% resolution) by optimizing the EB irradiation dose, as described in ref. [15] (SL.3). We find that the value of \( N_H \) estimated from the \( D/G \) peak ratios in Raman spectra (SL.3) is linearly proportional to the EB dose [Fig. 1(a)]. The non-local resistance \( (R_{NL}) \) as a function of the back gate voltage \( (V_G) \) is shown in Fig. 1(c) for hydrogenated graphene (H-graphene) with \( N_H < 0.06\% \) and using a conventional Hall-bar pattern. It reveals that a \( R_{NL} \) peak of \( \approx 40 \Omega \) appears around the Dirac point (inset). This result roughly agrees with previous reports using H-graphene fabricated by the same method, despite considering the contribution of the diffusion current. Samples without \( H \) do not exhibit the \( R_{NL} \) peak. This suggests that the observed \( R_{NL} \) peak in H-graphene is due to SOIs and spin Hall effects (SHEs) related to the C–H bond.

An optical micrograph and a schematic view of the four-probe electrode pattern (i.e., without a main current bar) used for the present main measurements are shown in Figs. 1(d) and 1(e), respectively. This probe pattern has been used to detect small-magnitude phase interference between electron spin waves as large changes in \( R_{NL} \) in lightly H-graphene. \( R_{NL} \) has been
measured between the 1–2 electrode pair as a function of \( V_{bg} \), while a fixed current flows between the 3–4 electrode pair. In this case, SHEs are generally difficult to be observed as \( R_{NL} \) peak at probes 3–4 due to the absence of a main Hall bar (which yields large spin current from electron current), while this absence allows effective spin confinement and accumulation around the \( R_{NL} \) probe site if spin current is caused from the fixed electron current flow at the 3–4 electrode pair. One expect that even a very small number of spins is confined and accumulates around the \( R_{NL} \) probe, resulting in a large spin density, which induces a phase interference path for WL. Hence, the WL associated with SOIs can be detected as evident changes in \( R_{NL} \).

The \( R_{NL} \) of \( H \)-graphene is shown as a function of \( V_{bg} \) and \( N_{H} \) in Fig. 2(a). As \( N_{H} \) increases, a small, broad \( R_{NL} \) peak appears around \( V_{bg} \sim 13 \) V. The peak dominates the very low \( N_{H} \) region (\(< 0.03\%\)). Its \( V_{bg} \) position roughly agrees with Fig. 1(c). In the crossover region (\( N_{H} \geq 0.03\% \)), this peak grows and two other peaks emerge at \( V_{bg} \sim 22 \) V and \( ~28 \) V. These behaviors are much evident in Fig. 2(b). The increase in \( R_{NL} \) peak values is from \( \sim 0.06R_{0} \) (where \( R_{0} = h/e^{2} \) is the resistance quantum) to \( 0.12R_{0} \) for \( N_{H} \) values increasing from 0% to \( \sim 0.06\% \). A similar correlation has been observed in at least five samples (SI.4).

The diffusion length decreases with increasing \( N_{H} \) as shown in Fig. 1(b), causing the diffusion current (arising from constant current flow between the 3–4 electrode pair) to also decrease around the \( R_{NL} \) probe. This decrease cannot explain the growth of \( R_{NL} \) in Figs. 2(a) and 2(b). Thus, one strong candidate of origins for the observed \( R_{NL} \) peaks is SOI (SHE) because only SOI (SH current) is induced with increasing \( N_{H} \) as shown in the inset of Fig. 4(c).

The in-plane \( B \)-dependence of the small \( R_{NL} \) peak around \( V_{bg} \sim 13 \) V is shown for \( N_{H} \geq 0.02\% \) in Fig. 2(c). The plot exhibits oscillatory behavior. This behavior agrees well with the Larmor spin-precession theory, as shown by the dotted calculation curve (\( \omega R = \Gamma \Delta B \leq D/W^{2} \), where \( \Gamma \) is the gyromagnetic ratio, \( \Delta B \) is the oscillation period, \( W \) is the width of the sample, and \( D \) is the spin diffusion coefficient). This agreement provides strong evidence for the presence of SOIs and SHEs. Our diffusive samples have low values of electron mobility \( \mu (\sim \sim 10,000\ cm^{2}/V^{*}s^{-1}) \), resulting in a small value of \( D/J/W^{2} \) (\( 4 \) \( \mu \) \( m_{u} \) \( \mu_{s} \)) but \( \mu \ll D_{s} \). However, the small value of \( \Delta B \) (\( \sim 0.2 \) T) for the data shown in Fig. 2(c) satisfies the condition \( \omega R = \Gamma \Delta B \leq D/J/W^{2} \) if we use the previously measured values of \( \Gamma \).

In contrast, the two other \( R_{NL} \) peaks (at \( V_{bg} \sim 22 \) V and \( ~28 \) V in Fig. 2(a)) have not been observed in \( H \)-graphenes with a conventional Hall-bar pattern [Fig. 1(b); SI.5]. Nor were these peaks observed in local resistance measurements at the 3–4 electrode pair (SI.6). Thus, these two \( R_{NL} \) peaks are unique to the present four-probe pattern measurement and this particular crossover regime (\( 0.03\% \leq N_{H} < 0.1\% \)). In this regime, other causes of spin accumulation, in addition to SOIs, are expected to dominate the \( R_{NL} \) peak.

To clarify the origin of the other \( R_{NL} \) peaks, the temperature (\( T \))-dependence of \( R_{NL}^{-1} \) has been measured in the crossover regime (\( N_{H} \geq 0.06\% \)). The semi-logarithmic \( T \)-dependence of non-local conductance (\( G_{NL} = R_{NL}^{-1} \)) for the three \( R_{NL} \) peaks in Fig. 2(a) is plotted in Fig. 3. The relationship is linear at high \( T \) (above \( ~6 \) K), and \( G_{NL} \) saturates below \( T = ~6 \) K. This behavior agrees qualitatively with that observed for WL in CNTs\(^{3} \), 2D electron-gas, and thin metal films [Fig. 1(a)] and suggests that the three \( R_{NL} \) peaks observed in Fig. 2(a) is dominated by WL rather than SOIs in the \( N_{H} \) crossover regime. This WL behavior on the \( T \)-dependence has not been observed at values of \( V_{bg} \) outside of those showing the three \( R_{NL} \) peaks (e.g., around the \( R_{NL} \) minimum; see SI.7). This further suggests that \( R_{NL} \) peaks only appear when the accumulated spin density arising from the SHE satisfies an optimal value that depends on \( V_{bg} \); when this condition is met, a constructive phase interference loop path for WL is created.

On the other hand, the critical temperature (\( T_{c} \)), which separates the high and low \( T \) regions, is extremely different from that of other 2D materials (SI.8). \( T_{c} \) \( = \sim 6 \) K at \( B = 0 \) T is considerably higher than the \( T_{c} \) observed in CNTs (e.g., \( T_{c} = 0.3 \) K). Above the \( T_{c} \) of conventional 2D materials, dephasing is dominated by inelastic scattering factors, while magnetic spin scattering (\( \tau^{-1} \); which is \( T \)-independent) dominates dephasing below the \( T_{c} \). Hence, a high \( T_{c} \) (like \( ~6 \) K) suggests a large \( \tau^{-1} \) rate. However, our graphene samples were mechanically exfoliated from graphite and contain no magnetic impurities.

Moreover, the \( R_{NL} \) peaks in Fig. 2(a) are strongly associated with SOIs, as shown in Fig. 2(c). Thus, the high \( T_{c} \) can be attributed to the SOI scattering rate \( \tau_{SOI}^{-1} \) instead of \( \tau^{-1} \). This result is consistent with ref. [7], which reported that SOIs act as a scattering center and suppress WL at values of \( B/\pi < 0.6 \). Because non-\( H \) graphene has a reported value of \( B/\pi = 0.5 \), the present \( H \)-graphene is expected to actually have a value of \( B/\pi < 0.6 \), when this theory is applicable..

The best fit (dashed lines in Fig. 3) of the WL theory [Eq. (1)] gives \( T_{c} \left( \tau_{SOI} \right) = 5.5 \) K and \( T_{c} \left( \tau_{c} \right) = 0.5 \) K by directly incorporating \( \tau_{SOI} \) into \( T_{c} \) (see SI.9). This quantitatively supports the strong contribution of SOIs to the high value of \( T_{c} \). The best fit to the linear \( T \)-dependence at high \( T \) also gives \( p = 4 \) for graphene [width of 4 \( \mu m \) and length of 24 \( \mu m \) in Fig. 1(e)]. The \( p \)-value suggests that electron–electron interactions cause dephasing in spin–phase interference (SI.8).

\[
G(T) = G_{0} + \frac{e^{2}}{2\pi^{2}\hbar} \left[ \frac{T}{T_{c}(\tau_{c})+T_{c}(\tau_{SOI})} \right]^{p}
\]

This result implies that \( B_{SOI} \) at low \( T \) suppresses dephasing arising from electron–electron interactions in WL. This also suggests that electron–electron scattering is dominant in the present system, while electron–phonon coupling is weak. Ref. [16] reported that SOIs increase the effective mass and decrease \( G \) under weak electron–phonon coupling. Our result
qualitatively agree with this. Moreover, ref [17] reported that a random Rashba field causes spin relaxation by spin-flip scattering, similar to an Elliott–Yafet (EY) mechanism. The value of $\tau^{-1}$ in this scenario is insensitive to $T$. In contrast, the D’yakonov-Perel (DP) mechanism becomes insensitive to $T$ only when electron-impurity scattering is dominant. In the present case, $\tau^{-1}$ is actually insensitive to $T$ in the low-$T$ region, as shown in Fig. 3. Electron-impurity scattering is not dominant because $N_H \ll 0.1\%$. Thus, the EY-like mechanism is dominant. When the Berry phase factor is considered, spin-flip scattering cannot occur even in the case mentioned above. In contrast, ref. [18] reported that coupling of Dirac fermions to phonons introduces new structures in their dispersion curves and, in the case of massive Dirac fermions, can shift and modify the gap. The present weak electron-phonon coupling case does not correspond to this and, hence, the SOI gap cannot be modified.

$T_c$ increases with increasing $N_H$ (inset of Fig. 3). This suggests SOI-suppressed dephasing because the strength of SOIs increases with $N_H$ [inset of Fig. 4(c)]. The SOI relaxation length ($L_{SOI}$) $\sim 2 \mu$m at $T = T_c$ can be estimated from the $T$-dependence under a high external magnetic field ($B = 7$ T), as shown in Fig. 3 (see also SI.8). Importantly, the presence of only positive conductance and its saturation at low temperatures suggest the absence of AL at $T > 1.5$ K. This supports the theory$^7$ that SOIs do not cause spin-flipping in graphene at $T > 1.5$ K.

When the application of an external magnetic field perpendicular $B$ ($B_z$) to the graphene plane was applied, it directly demonstrates the abovementioned SOI-based suppression of dephasing in WL, and validates the absence of spin-flipping for AL. In the presence of $B_z$, negative MRs for the three $R_{NL}$ peaks at $N_H = 0.06\%$ in Fig. 2(a) are observed around $B = 0$ and $T = 1.5$ K (Fig. 4(a) for the $R_{NL}$ peak at $V_{bg} \sim 22$ V; see SI.11 for the other two peaks). Because the negative MR denotes a decrease in MR due to dephasing in constructive phase interference with increasing $B_z$ (Fig. 4(e)), it indicates the presence of WL. The dotted red curve in Fig. 4(a) is a result calculated from the WL-based theory$^{12}$, considering SOI-suppressed dephasing via small $L_{SOI}$ (SI.10; $L_{SOI} \sim 2 \mu$m for strong SOI obtained from Fig. 3). It actually shows good agreement with the experimental curve. In contrast, the dotted blue curve obtained using $L_{SOI} \sim 10 \mu$m (i.e., weak SOI) exhibits a significantly sharper trend and deviates from the experimental curve. This suggests that the negative MR is associated with SOIs. Indeed, $T = 1.5$ K for this negative MR observation corresponds to the low-$T$ regime ($T < T_c$) within SOI-suppressed dephasing in Fig. 3.

The negative MR amplitude (i.e., the ratio of MR before and after the drop) in WL is reduced to $\sim 25\%$ when $N_H$ increases from 0% to 0.1%, as shown by Fig. 4(b) and the black symbols in Fig. 4(c). An increase in $N_H$ actually induces SOIs [inset of Fig. 4(c)]; a reduction in $L_{SOI}$ is estimated by fitting the data in Fig. 4(b) to the WL theory (SI. 10). These results strongly suggest that $B_{SOI}$, due to the induced SOIs, suppresses the MR drop ratios (Fig. 4(e); i.e., suppression of dephasing in WL) and tends to stabilize the spin moment against an externally applied $B_z$ (i.e., by screening $B_z$). These results also imply the absence of spin-flipping and AL, even with SOIs (SI(2)) at $T > 1.5$ K. These conclusions are consistent with Fig. 3, and support the hypothesis that $B_{SOI}$ suppresses dephasing in WL in the presence of $B_z$. Negative MRs can only be observed in the three $R_{NL}$ peaks shown in Fig. 2(a) (SI. 11). This is consistent with the result of $T$-dependence in the WL (Fig. 3 and SI.7).

MRs observed around the three $R_{NL}$ peaks for the sample in Fig. 2(a) ($N_H = 0.06\%$) for $T = 1.5$ K and in the presence of an external in-plane magnetic field $B_y$ are shown in Fig. 4(d) (SI.11). In contrast to the case of $B_z$ [Figs. 4(a)–(c)], $B_y$ basically does not cause dephasing and does not change MR, although there is an exception for the case in which a component of $B_y$ is transferred to $B_z$ by a strong ripple; this is caused by substrate roughness in graphene$^{14}$. Because our sample does not correspond to this, the observed negative MRs are anomalous. The behavior of MR is approximately linear and drastically different from the oscillatory behavior due to SOI in the low-$N_H$ region ($\sim 0.03\%$) (Fig. 2(c)). Only when $V_{bg}$ is 22 V, MR exhibits an oscillatory behavior with a period of $ABV \sim 4$ T. As mentioned for Fig. 2(c), our diffusive samples have low values of $\mu$ and $W = 4 \mu$m, leading to a small value of $D/V^2$. The $AB$ value of $\sim 4$ T cannot satisfy the condition $\omega \tau \ll \Gamma \Delta B_y \lesssim D/V^2$, assuming similar values of $\Gamma$, and cannot be interpreted in terms of the Larmor spin precession. Consequently, the Larmor spin precession is not the dominant effect in the crossover regime ($N_H \lesssim 0.06\%$) shown in Fig. 4(d).

Because WL dominates in the crossover region, the linear MR relationships are also associated with SOI-based suppression of dephasing in WL as mentioned above. It can be understood by a theory that OP-symmetric SOIs are linearly reduced by coupling with the in-plane Zeeman effect by applying an in-plane $B_z$. Subsequently, the SOI-based suppression of dephasing is also reduced and the dephasing recovers$^2$. In our samples, the dephasing in WL is suppressed by SOIs even at $B = 0$ and $T = 1.5$ K (i.e., $T < T_c$), as explained for Fig. 3. Thus, the MR value at $B = 0$ is larger than that in the case for no suppression. With increasing $B_z$, SOIs linearly decrease by coupling with the in-plane Zeeman effect. Subsequently, the SOI-triggered suppression of dephasing is also linearly reduced, resulting in the recovery of dephasing and a linear decrease in the MR values. The dotted red lines in Fig. 4(d) are the best fits to the experimental data based on this theory$^6$. Indeed, the fits are reasonably good. Consequently, the $B_z$-dependence reconfirms SOI-based suppression of dephasing in WL and the observed absence of AL.

In conclusion, we observed that Rashba-type SOIs in graphene can suppress dephasing in WL without causing spin-flipping (AL), because the SOIs are OP-symmetric. SOIs were introduced to graphene by light hydrogenation using precisely controlled EB irradiation doses. In the crossover regime ($N_H \sim 0.06\%$), the coexistence of SOIs with WL was confirmed by three $R_{NL}$ peaks. The $T$-independent $R_{NL}$ peak was caused by the random Rashba $B_{SOI}$ at low $T$, which acted as a magnetic scattering center. This suppressed the dephasing caused by electron–electron scattering in WL and led to values of $T_c$, as high as $\sim 6$ K. The magnitude of $B_z$-dependent dephasing in the negative MR was suppressed to $\sim 25\%$ due to the $B_{SOI}$ induced by

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increasing \( N_H \) (from 0\% to 0.1\%). On the other hand, the suppression was reduced by restoring WL through coupling of the SOIs with in-plane Zeeman splitting under \( B_c \). Ref. [19] reported that the effect of a hexagonal warping term in the Hamiltonian of TI electronic states changes both the orbital magnetic moment and its OP spin texture in the Berry phase. The Fermi surface average of the \( z \)-component of spin is also closely related to the value of the Berry phase. These results are strongly associated with our own findings. The research presented herein shows that lightly hydrogenated graphene can be used to realize TI states and promises novel characteristics in SOI-triggered graphene spintronic devices.

The authors thank T. Enoki, M. Yamamoto, S. Tarucha, H. Shinohara, T. Ando, H. Hibino, B. Ozilmmaz, S. Roche, A. H. Macdonald, P. Seneor, R. Wiesendanger, and M. S. Dresselhaus for their technical contribution, fruitful discussions, and continuous encouragement.

The work at Aoyama Gakuin was partly supported by a Grant-in-aid for Scientific Research (Basic research A: 24241046) in MEXT and AOARD grant (135049) in U.S. Air Force Office of Scientific Research. The Tokyo University’s work was also supported by the Special Coordination Funds for Promoting Science and Technology.

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8. M. Wojtaszek et al., Phys. Rev. B 87, 081402(R) (2013): This was highly valuable for spintronic devices. However, the origin was not clarified because the result was controversial to conventional cases understandable by Elliott-Yafet (EY) or D’yakonov-Perel (DP) mechanisms for spin scattering (SI (8)), in which an increase in SOI reduces spin relaxation length and time.
Figure 1
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Figure 2

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Figure captions

Fig. 1
(a) Volume (area) of $N_H$, estimated from the D/G peak ratios in Raman spectra, as a function of the EB dose incident on an HSQ resist on graphene (SI.3). Error bars have been shown for three – five graphenes through all figures.
(b) Diffusion length estimated from WL measurements as a function of $N_H$.
(c) Non-local resistance ($R_{NL}$) as a function of back gate voltage ($V_{bg}$) for a sample with $N_H = 0.06\%$ and using a conventional Hall-bar pattern. Fixed currents of 100 nA are applied at both the main Hall bar and the $R_{xy}$ probes. $R_{NL}$ is measured by another $R_{xy}$ probe that is placed 3 $\mu$m away from the first. Inset: Dirac point observed in the sample.
(d) Optical micrograph of the four-probe electrode pattern used for the present main measurements.
(e) Schematic view of (d) with electron wave trajectories for SHE and WL. Fixed currents of 100 nA are flown at the 3–4 electrode pair and $R_{NL}$ is measured at the 1–2 electrode pair.

Fig. 2
(a) $R_{NL}$ of $H$ graphene as a function of $V_{bg}$ and $N_H$ using the electrode pattern shown in Fig. 1(d) and 1(e). $V_{bg}$ is swept from 0 V to $+V_{bg}$ to avoid hole-doping through the $p$-type substrate. Electrons cannot be injected from surface hydrogen adatoms when $N_H < 0.1\%$. The $N_H$ values of some samples were tuned by detaching $H$ atoms using high-temperature annealing in a high vacuum.
(b) $R_{NL}$ vs. $N_H$ for the three $R_{NL}$ peak values observed in (a).
(c) In-plane $B$-dependence of the $R_{NL}$ peak at $V_{bg} = 13$ V for the sample with $N_H = 0.02\%$ in (a). The dotted line is the best fit to the theory of Larmor spin precession.

Fig. 3
Semi-logarithmic temperature dependence of the inverse values of the three $R_{NL}$ peaks in Fig. 2(a) for a perpendicular $B = 0$ and 7 T. The sample is in the crossover regime ($N_H = 0.06\%$). The dashed lines are the best fit by the WL formula directly considering the influence of SOI ($\delta_{SOI}$) on $T_c$ (Eq. (1)). These WL behaviors are observed only around the three $R_{NL}$ peaks in Fig. 2(a) (see SI (7), which explains the absence of this relationship in $R_{NL}$ minima).
Inset: $T_c$ between the linear (high $T$) and saturation (low $T$) regions of the main panel as a function of $N_H$.

Fig. 4
(a) $R_{NL}$ vs. the perpendicular magnetic field $B_{\perp}$ at $V_{bg} \sim 22$ V for the sample with $N_H = 0.06\%$ in Fig. 2(a). The red and blue dotted curves are fits to the data by the WL-related theory$^{12}$ using strong and weak SOI parameters ($L_{SOI} = 2 \mu$m and 10 $\mu$m, respectively) in the expression for the magnitude of the MR drop (SI.10).
(b) $R_{NL}$ vs. $B_{\perp}$ at $V_{bg} \sim 22$ V for samples with different values of $N_H$. The difference in MR$_{NL}$ drop ratios is given by $\Delta MR_{NL}/MR_{NL}(B = 0)$. MR$_{NL}$ values of individual curves are overlapped at $B = 0$ to directly compare the MR$_{NL}$ drop ratios.
(c) $R_{NL}$ peak values at $B = 0$ (open symbols for the left axis) and MR-drop amplitude $[\Delta MR_{NL}/MR_{NL}(B = 0)]$ (black symbols for the right axis) in WL as a function of $N_H$. $\Delta MR_{NL}$ is defined as the difference between the values of MR$_{NL}$ at the peak ($B_{\perp} = 0$) and at the bottom ($B_{\perp} = +2T$) of individual curves in (b).
Inset: Best-fit $L_{SOI}$ values obtained from the individual curves in Fig. 4(b) using the WL theory in (a) (SI(10)).
(d) $R_{NL}$ vs. $B_{\perp}$ for the three peaks at $N_H = 0.06\%$ of Fig. 2(a). The dotted lines represent best fits to the data using the theory for linear reduction of SOI-suppressed dephasing (i.e., linear recovery of the dephasing) in WL by the coupling of SOI with the in-plane Zeeman effect.$^6$